PMM U.S.S.R.,Vol.46,pp.261-267
0021-8928/83/2 0261 $7.50/0

UDC 539.3

ON THE THEORY OF COUPLED LOSS OF STABILITY IN STIFFENED THIN-WALLED STRUCTURES*

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A system of the principal nonlinear approximation equations is obtained for the problem of loss of stability is stiffened thin-walled structures in the presence of finite displacements, taking into account the presence of a set of local modes with critical loads differing little from each other. A concept of "modified" local modes is proposed, allowing an estimation of the mode interaction already in the first nonlinear approximation. The possibility of simplification of the final system of equations, taking into account the fact that the local mode length is short compared with that of the overall mode, is shown. It is established that within the framework of the principal nonlinear approximation every local mode in the stiffened plates and shells interacts with the overall mode, but there is no explicit interaction between the local modes themselves. A theorem is proved establishing the correlation between the system with one, and with many local modes. The problem of stability of a compressed stiffened plate, i.e. of a wide strut, is solved as an example. The proposed theory can be applied to structures almost equally stable when no local waves form up to the moment of coupled buckling.

The study of coupled buckling modes in stiffened plates and shells engaged in recent years the attention of a number of authors /1,2/ give the bibliography. Every one of them, however, dealt only with the interaction of two buckling modes, the overall and the local mode. Separation of a single overall mode is justified since the spectrum of overall modes is usually scarce. The local modes, however, usually have short wave lengths and a sufficiently dense spectrum. A question arises, whether the inclusion of a still larger number of interacting local modes will not lead to increasingly larger reduction in the value of the critical load. The question of an efficient method of solving the problem of coupled buckling is also important. From the approximate solution of the problem for a stiffened plate and cylindrical shell /2 – 4/ and the general qualitative analysis /5/ it follows that the interaction between the overall and local modes in stiffened structures becomes dangerous only when the general flexure takes place in one of two possible directions (relative to the stiffeners). Therefore, in the presence of the general oscillatory wave formation the coupled buckling zones must alternate with the single-mode buckling zones where no waves form locally. On changing the sign of the overall flexure, every zone is expected to pass from the single-mode buckling branch to the coupled buckling branch and vice versa, and this passage leads to kinks in the resulting load-displacement curves. Non-differentiability at the salient point makes the asymptotic Koiter method /6, 7/ based on expanding the load into a power series in terms of the displacement amplitudes, inefficient. When this method is used /8/ to solve a problem for a stiffened shell, all third order terms in displacements and their derivatives, appearing in the potential energy term, vanish by virtue of the periodicity and symmetry, and the effect of mode interaction is discovered only when the problem is solved in a higher degree of approximation.

The Ritz method is found suitable for solving the problem of coupled buckling. The displacement field is given, with due regard to the manner of wave formation described above, in the form of a sum of the overall mode and the modified local modes of the linear problem $u_i^*: u_i^* = u_i$ or $u_i^* = 0\ (i > 1)$ depending on the sign of the overall displacement. The condition of absence of the discontinuities in the force and deformation factors demand that an additional field u_i^{**} appearing near the nodes of the overall mode is imposed on the field u_i^*. This state should have the character of a boundary layer and its contribution towards the overall energy of the system can be neglected, since the length of the local mode is short compared with the overall mode. Such an approach, used in the approximate solution /2,4/ is close to the "mode modulation" method proposed by Koiter in /5/ and makes possible the investigation of the mode interaction already in the first nonlinear approximation since the third order terms in the energy functional are preserved.

*Prikl.Matem.Mekhan.,46,No.2,pp.337-345,1982

1. Basic equations. Let us consider a stiffened structure for which the overall and local buckling modes can be separated (under the local mode we understant the buckling during which there is no transverse displacement of the sheathed stiffeners). It is assumed that the spectrum of the eigenvalues for the overall modes is sufficiently scarce, so that we can limit ourselves to considering a mode with the smallest eigenvalue λ_1. Let us assume that n local modes can be singled out, with the eigenvalues λ_s $(s = 2, \ldots, n+1)$ close to λ_1. We construct the energy functional of the system by separating the principal linear and nonlinear terms, using the compact notation and a number of results of the general asymptotic theory of stability /6,7/. Let u, ε, σ denote the tensor translation, deformation and stress fields. The primary stress-deformation state determined by the equations of the linear theory of elasticity is assumed to depend linearly on the load parameters λ (of the field λu_0, $\lambda\varepsilon_0$, $\lambda\sigma_0$). The geometrical and elasticity relations can be written in the form

$$\varepsilon = l_1(u) + {}^1/_2\, l_2(u), \quad \sigma = H(\varepsilon) \tag{1.1}$$

where l_1 and H are linear operators and l_2 is a quadratic operator. Denoting by $\sigma\cdot\varepsilon$ the work done by the stress field σ on the deformation field ε, we write the potential energy of the system in the form

$$\Phi = {}^1/_2\, \sigma\cdot\varepsilon - \lambda\sigma_0\cdot l_1(u) \tag{1.2}$$

$$\left(\sigma\cdot\varepsilon \equiv \int\limits_V \sigma_{ij}\varepsilon_{ij}\, dV = \int\limits_S (N_x\varepsilon_x + 2N_{xy}\varepsilon_{xy} + N_y\varepsilon_y + M_x\chi_x + 2M_{xy}\chi_{xy} + M_y\chi_y)\, dS\right)$$

where the second term is equal to the work done by external forces expressed in terms of the internal forces, with the state of equilibrium σ_0 taken into account. Repeated indices denote summation.

Let us assume that a linear problem of stability has been solved and the eigenvalues λ_i together with the forms u_i $(i = 1, 2, \ldots, n+1)$ found. The forms are assumed to be normalized in some manner, by e.g. the condition that the maximum displacements are equal to the shell thickness. The orthogonality (relative independence) of these forms is expressed by the condition

$$\sigma_0\cdot l_{11}(u_i, u_j) = 0 \ (\text{ or } \ \sigma_i\cdot l_1(u_j) = 0), \ i \neq j \tag{1.3}$$

where l_{11} is a bilinear operator associated with the operator

$$l_2(u)\ l_2(u+v) = l_2(u) + 2l_{11}(u, v) + l_2(v)\ l_{11}(u, u) = l_2(u)$$

In the nonlinear problem we write the displacement field in the form

$$u = \lambda u_0 + \xi_1 u_1 + \xi_s u_s^* \tag{1.4}$$

$$u_s^* = u_s E(\pm\xi_1), \qquad E(x) = \begin{cases} 1, & x \geqslant 0 \\ 0, & x < 0 \end{cases}$$

Here and in what follows, the repeated index s (and also r) denotes summation from 2 to $n+1$, and the indices i, j, k the summation from 1 to $n+1$. The sign of ξ_1 in the unit function E is assigned in the course of solving the problem.

According to (1.1) the displacements (1.4) have the corresponding deformation and stress fields (the asterisk accompanying u_s will, from now on, be omitted for simplicity)

$$\varepsilon = \lambda e_0 + \xi_i e_i + \xi_i\xi_j\varepsilon_{ij}, \quad \sigma = \lambda\sigma_0 + \xi_1\sigma_i + \xi_i\xi_j\sigma_{ij} \tag{1.5}$$

$$(e_i = l_1(u_i), \ \ \varepsilon_{ij} = {}^1/_2\, l_{11}(u_i, u_j), \ \ \sigma_i = H(e_i), \ \ \sigma_{ij} = H(\varepsilon_{ij}))$$

Substituting (1.4) and (1.5) into (1.2) and using the relation expressing the mutuality of the works $H(\varepsilon_i)\cdot\varepsilon_j = H(\varepsilon_j)\cdot\varepsilon_i$, we obtain the following expression for the energy (the fourth order terms in ξ_i are neglected, and this corresponds to the domain of applicability of the expression (1.4)):

$$2\Phi = -\lambda^2\sigma_0 e_0 + \xi_i\xi_j[\lambda\sigma_0\cdot l_{11}(u_i, u_j) + \sigma_i e_i] + \xi_i\xi_j\xi_k\sigma_i l_{11}(u_j, u_k) \tag{1.6}$$

The assumption that the wave lengths of the local modes are small compared with the overall mode, enables us to simplify the expression (1.6). The mutual cancellation of the integrals over the segments with different direction of the local flexure (within the limits of a single half-wave of the overall mode), implies that all terms of the functional Φ depending on the odd power of the displacements of the local mode either vanish, or become negligibly small

$$\sigma_i\cdot l_{11}(u_j, u_k) = 0 \tag{1.7}$$

except in cases when one of the indices is equal to unity and the other two coincide. According to this rule we can also accept as valid the orthogonality relations (1.3) for the modified local modes u_s^*. Taking this into account, we obtain the expression (1.6) for the energy in the form

$$\Phi = a_0 + \frac{1}{2} a_i \left(1 - \frac{\lambda}{\lambda_i}\right) \xi_i^2 + \frac{1}{2} b_i \xi_1 \xi_i^2 - \frac{\lambda}{\lambda_i} a_i \bar{\xi}_i \xi_i \tag{1.8}$$

$$a_0 = -{}^1/_2 \lambda^2 \sigma_0 \cdot l_0, \quad a = -\lambda_i \sigma_0 \cdot l_2(u_i), \quad b_i = \sigma_1 \cdot l_2(u_i) + 2\sigma_i \cdot l_{11}(u_1, u_i)$$

The last term in (1.8), which describes the influence of the initial imperfections over all forms $\bar{\xi}_i$, is introduced in accordance with the results of the general theory /6,7/ (the field of initial imperfections is assumed to have the form $\bar{u} = \bar{\xi}_i u_i$).

The following eigenvalues of the linear problem were used in (1.8):

$$\lambda_i = -\sigma_1 \cdot l_1(u_i) / \sigma_0 \cdot l_2(u_i)$$

The above relations are obtained from (1.6) by leaving in the expression for Φ the quadratic terms only. They contain the modified local modes u_i, but the specific features of the local modes imply that the values λ_i can be assumed as equal to the exact eigenvalues of the linear problem, i.e. u_i in these formulas can be regarded as the exact linear modes. We note that if the overall mode is symmetric and periodic if only in a single direction, then $\sigma_1 \cdot l_2(u_1) = 0$ identically (e.g. in the case of stiffened plates when more than one half-wave form in the longitudinal or transverse direction, and in the stiffened shells). Then the expression for the energy (1.8) contains no terms with ξ_1^3.

The conditions of stationarity of the potential energy lead to the following system of nonlinear equations (summation over s is not implied):

$$a_1 \left(1 - \frac{\lambda}{\lambda_1}\right) \xi_1 + \frac{1}{2} \sum_{j=1}^{n+1} b_j \xi_j^2 = \frac{\lambda}{\lambda_1} a_1 \bar{\xi}_1 \tag{1.9}$$

$$a_s \left(1 - \frac{\lambda}{\lambda_s}\right) \xi_s + b_s \xi_1 \xi_s = \frac{\lambda}{\lambda_s} a_s \bar{\xi}_s$$

The structure of the expression (1.8) for Φ which does not include the products $\xi_i \xi_s$ $(r, s > 1)$ shows, that under the assumptions (1.7) adopted the local modes do not interact explicitly with each other (the implicit interaction manifests itself through the interaction of every local mode with the overall mode). The systems in which one mode interacts with the remaining modes, the latter not interacting explicitly with each other, can be described as systems with simple interaction. The stiffened plates and shells refer, within the limits of the principal nonlinear approximation, to systems of this type.

The condition of stability of the equilibrium branch is represented by the positive definiteness of the second variation of the potential energy, and this requires that the Jacobian of the system (1.9) be positive

$$J = \left[\Lambda_1 - \sum_{s=2}^{n+1} (b_s \xi_s)^2 \Lambda_s^{-1}\right] \prod_{r=2}^{n+1} \Lambda_r \tag{1.10}$$

$$\Lambda_j \equiv a_j \left(1 - \frac{\lambda}{\lambda_j}\right) + b_j \xi_1, \quad j = 1, 2, \ldots, n+1$$

The Jacobian J vanishes at the limit point corresponding to maximum load, and at the points of bifurcation. The equality to zero of every Λ_r under the product sign corresponds, as the second equation of (1.9) implies, to the bifurcation point, in the absence of the initial deflection over the corresponding mode $\bar{\xi}_r = 0$, and we have $\lambda/\lambda_r = 1 + b_r \xi_1 / a_r$. If all $\bar{\xi}_r = 0$ then, as we see from the second equation of (1.9) every Λ_r appearing under the product sign no longer vanishes and the condition that the Jacobian be zero, yields the relation

$$\Lambda_1 - \sum_{s=2}^{n+1} (b_s \xi_s)^2 \Lambda_s^{-1} = 0 \tag{1.11}$$

The sign of the coefficients b_s in the equations obtained depends on the choice of sign in the unit function $E(\pm\xi_1)$ when the local modes in (1.4) are modified. The sign is chosen from the condition $b_s \xi_1 < 0$ (only then the interaction of the s-th mode with the overall mode leads to reduction in the value of the limiting load).

2. Analysis of the solution. When the initial deflection occurs only in the overall mode ($\bar{\xi}_1 \neq 0$, $\bar{\xi}_s = 0$) two types of solution of (1.9) exist:

a) $\xi_s = 0$ $(s = 2, 3, \ldots, n+1)$, $\xi_1 \neq 0$,

i.e. the equilibrium branch is situated in the space $\lambda, \xi_1, \ldots, \xi_{n+1}$ on the plane $\lambda - \xi_1$, b) all ξ_i are different from zero. The corresponding branches are naturally called noncoupled and coupled (we note that the overall buckling can be noncoupled, but the local one cannot). When the initial deflection over the local modes is present, only coupled buckling is possible (this follows from the second equation of (1.9)).

Let us consider, for the general case, the limit point $\lambda = \lambda_*$ on the equilibrium branch described by the system of equations (1.9), (1.11). We shall limit ourselves, for simplicity, to the case when $b_1 = 0$ (symmetric characteristics on the overall mode). Eliminating ξ_s from the above system we can write it in the form

$$F_1(\lambda, \xi_1) \equiv a_1\left(1 - \frac{\lambda}{\lambda_1}\right)\xi_1 - \sum_{s=2}^{n+1} P_s(\lambda, \xi_1)\,\bar{\xi}_s^2 - \frac{\lambda}{\lambda_1} a_1 \bar{\xi}_1 = 0 \tag{2.1}$$

$$F_2(\lambda, \xi_1) \equiv a_1\left(1 - \frac{\lambda}{\lambda_1}\right) - \sum_{s=2}^{n+1} Q_s(\lambda, \xi_1)\,\bar{\xi}_s^2 = 0$$

$$P_s(\lambda, \xi_1) \equiv -\frac{1}{2} b_s \left(\frac{\lambda}{\lambda_s} a_s \Lambda_s^{-1}\right)^2, \quad Q_s(\lambda, \xi_1) = \left(\frac{\lambda}{\lambda_s} b_s a_s\right)^2 \Lambda_3^{-3}$$

The structure of system (2.1) enables us to prove, under a single assumption concerning the functions $Q_s(\lambda, \xi_1)$ formulated below, the following theorem which establishes a definite relationship between the system with a single, and with n local modes.

Theorem. The limiting value λ_* for a system with n local modes is not smaller than the limiting value $\lambda_*^{(k)}$ of a system with a single, most dangerous local mode, with the initial deflection $\bar{\xi}^0 = \sqrt{n}\,\bar{\xi}_M$, where $\bar{\xi}_M = \max_s \bar{\xi}_s$.

We shall call $\bar{\xi}^0$ the equivalent deflection. Under the most dangerous local mode we shall understand the mode (with the index $s = k$) for which the limiting load under an equivalent deflection will be smallest (for all s). The values $\lambda_*^{(s)}$ for every s and the corresponding displacements $\xi_1^{(s)}$ can be found from

$$F_1^{(s)}(\lambda, \xi_1) \equiv a_1\left(1 - \frac{\lambda}{\lambda_1}\right)\xi_1 - P_s(\lambda, \xi_1)\, n\bar{\xi}_M^2 - \frac{\lambda}{\lambda_1} a_1 \bar{\xi}_1 = 0 \tag{2.2}$$

$$F_2^{(s)}(\lambda, \xi_1) \equiv a_1\left(1 - \frac{\lambda}{\lambda_1}\right) - Q_s(\lambda, \xi_1)\, n\bar{\xi}_M^2 = 0$$

Let us first note the following properties of the quantities entering the equations (2.1) and (2.2):

1) the following inequalities hold (we consider the limiting load lying below the bifurcation points): $a_s > 0$, $\Lambda_s > 0$;

2) we can assume without loss of generality that

$$\bar{\xi}_1 > 0,\ \xi_1 > 0,\ b_s < 0\ (b_s \xi_1 < 0); \tag{2.3}$$

3) $P_s(\lambda, \xi_1)$ and $Q_s(\lambda, \xi_1)$ under the conditions (2.3) increase monotonically in each of their arguments;

4) $F_1(\lambda, \xi_1)$ is a monotonically increasing function for the values of λ lying on the stable segments of the equilibrium branch. This follows from the fact that, as we see from (2.1),

$$\partial P_s(\lambda, \xi_1)/\partial \xi_1 = Q_s(\lambda, \xi_1),\quad \partial F_1(\lambda, \xi_1)/\partial \xi_1 = F_2(\lambda, \xi_1) \tag{2.4}$$

and the function $F_2(\lambda, \xi_1)$ is positive in the interval $0 < \lambda < \lambda_*$. An analogous assertion holds for $F_1^{(s)}(\lambda, \xi_1)$;

5) $F_2^{(s)}(\lambda, \xi_1)$ and $F_2(\lambda, \xi_1)$ are monotonically decreasing function of ξ_1 and λ. This follows from the properties 2) and 3).

To prove the theorem, we write the second equation of (2.2) in the form

$$\frac{\lambda_*^{(s)}}{\lambda_1} = 1 - \frac{n\bar{\xi}_M^2}{a_1} Q_s[\lambda_*^{(s)}, \xi_1^{(s)}(\lambda_1^{(s)})] \tag{2.5}$$

(the argument accompanying ξ_1 indicates that this quantity is connected with the value $\lambda_*^{(s)}$ by the first equation (2.2)). For the k-th mode with the smallest $\lambda_*^{(k)}$ the quantity $Q_k(\lambda_*^{(k)}, \xi_1^{(k)})$ must have its maximum value with respect to s

$$Q_k[\lambda_*^{(k)}, \xi_1^{(k)}(\lambda_*^{(k)})] \geqslant Q_s[\lambda_*^{(s)}, \xi_1^{(s)}(\lambda_*^{(s)})],\quad s = 2, 3, \ldots, n+1 \tag{2.6}$$

We shall now assume that the above condition holds not only for the limit points, but also for all λ lying on the stable segment of the equilibrium branch in accordance with the first equation of (2.1), i.e. when $0 \leqslant \lambda < \lambda_*$ and $0 \leqslant \xi_1 \leqslant \xi_1(\lambda_*)$

$$Q_k(\lambda, \xi_1) \geqslant Q_s(\lambda, \xi_1)\ \ (s = 2, 3, \ldots, n+1;\ 0 \leqslant \lambda < \lambda_*) \tag{2.7}$$

Integrating the inequality (2.7) with respect to ξ_1 from 0 to ξ_1 corresponding, in accordance with the first equation of (2.1) to some value of λ, and taking (2.5) into account, we obtain

$$P_k[\lambda, \xi_1(\lambda)] \geqslant P_s[\lambda, \xi_1(\lambda)] \ (s = 2, 3, \ldots, n+1, \ 0 \leqslant \lambda \leqslant \lambda_*) \tag{2.8}$$

From (2.8) and (2.7) it follows that for $0 \leqslant \lambda < \lambda_*)$ we have

$$\sum_{s=2}^{n+1} P_s[\lambda, \xi_1(\lambda)]\bar{\xi}_s^2 \leqslant nP_k[\lambda, \xi_1(\lambda)]\bar{\xi}_M^2 \tag{2.9}$$
$$\sum_{s=2}^{n+1} Q_s[\lambda, \xi_1(\lambda)]\bar{\xi}_s^2 \leqslant nQ_k[\lambda, \xi_1(\lambda)]\bar{\xi}_M^2$$

Equating the left-hand sides of the first and second equations of (2.1) and (2.2), respectively, we obtain the following relations for the points lying on the equilibrium branch:

$$F_1[\lambda, \xi_1(\lambda)] \geqslant F_1^{(k)}[\lambda, \xi_1(\lambda)] \tag{2.10}$$
$$F_2[\lambda, \xi_1(\lambda)] \geqslant F_2^{(k)}[\lambda, \xi_1(\lambda)]$$

Since on the equilibrium branch $F_1(\lambda, \xi_1) = 0$, it follows from the first inequality of (2.9) and property 4), that the displacement $\xi_1^{(k)}$ determined for a given λ from the first equation of (2.2), cannot be smaller than $\xi_1 : \xi_1^{(k)} \geqslant \xi_1$. But if we replace in the right-hand side of the second inequality of (2.10) ξ_1 by $\xi_1^{(k)}$ then, according to the property 5) the inequality will become stronger

$$F_2[\lambda, \xi_1(\lambda)] \geqslant F_2^{(k)}[\lambda, \xi_1^{(k)}(\lambda)] \tag{2.11}$$

For all $\lambda < \lambda_*^{(k)}$ we find that when the right-hand side of this inequality is greater than zero, so is the left-hand side, i.e. the limit value λ_* which makes $F_2[\lambda, \xi_1(\lambda)]$ equal to zero cannot be smaller than $\lambda_*^{(k)}$, and this proves the theorem.

The assumption used in the proof, which enabled the passage from the inequality (2.6) to (2.7) to be made, was required to justify (2.9). When this assumption fails, a weaker theorem can be proved in which the equivalent deflection is understood to be

$$\bar{\xi}^0 = \sqrt{n}\,\bar{\xi}_M\psi, \quad \psi = \left\{\frac{\max_s P_s[\lambda_*, \xi_1(\lambda_*)]}{P_k[\lambda_*, \xi_1(\lambda_*)]}\right\}^{1/2}$$

Then the first inequality of (2.9) (and other inequalities following from it) will hold at $\lambda = \lambda_*$ irrespective of whether (2.8) holds or not. The results of solving concrete systems (some of them are given below) show that when the conditions (2.7), (2.8) are violated, the theorem remains valid in its initial form. This is explained by the fact that the inequality (2.11) obtained by consecutive strengthening of the series of inequalities remains valid when (2.7) is violated, provided that the magnitude of ψ does not appreciably exceed unity.

The proved theorem can be used to reduce the systems with n local modes to a system with a single local mode. In a particular case when the deflections are the same for all modes, the equivalent deflection is found to be $\sqrt{n}$ times smaller than the sum of the amplitudes of all deflections at $\psi = 1$.

3. Interaction of two modes. When a single local mode with a symmetric characteristic on the overall mode ($b_1 = 0$) is considered, it is expedient to perform the following transformation (the subscript 2 denotes the local mode):

$$\xi_1 = -\frac{z_1}{d_1}, \quad \xi_2 = \frac{z_2}{\sqrt{d_1 d_2}}, \quad \bar{\xi}_1 = -\frac{\bar{z}_1}{d_1}, \quad \bar{\xi}_2 = \frac{\bar{z}_2}{\sqrt{d_1 d_2}}, \quad d_1 = \frac{b_2}{a_2}, \quad d_2 = \frac{b_2}{a_1} \tag{3.1}$$

The potential energy Φ (1.8) and the system of equations of equilibrium (1.9) assume the following form in the new variables:

$$\Phi^\circ \equiv \Phi\frac{b_2^2}{a_1 a_2^2} = \frac{1}{2}\left(1 - \frac{\lambda}{\lambda_1}\right)z_1^2 + \frac{1}{2}\left(1 - \frac{\lambda}{\lambda_2}\right)z_2^2 - \frac{1}{2}z_1 z_2^2 - \frac{\lambda}{\lambda_1}\bar{z}_1 z_1 - \frac{\lambda}{\lambda_2}\bar{z}_2 z_2 \tag{3.2}$$

$$\left(1 - \frac{\lambda}{\lambda_1}\right)z_1 - \frac{1}{2}z_2^2 = \frac{\lambda}{\lambda_1}\bar{z}_1, \quad \left(1 - \frac{\lambda}{\lambda_2}\right)z_2 - z_1 z_2 = \frac{\lambda}{\lambda_2}\bar{z}_2 \tag{3.3}$$

The condition that the Jacobian is equal to zero is reduced, for the limit or birfurcation point (1.11) to

$$\left(1 - \frac{\lambda}{\lambda_1}\right)\left(1 - \frac{\lambda}{\lambda_2} - z_1\right) - z_2^2 = 0 \tag{3.4}$$

The potential energy is even in z_2 (semisymmetric systems). Equations of the type (3.3) (written in the variables $\xi_i, \bar{\xi}_i$) were studied earlier in connection with the problems of buckling in smooth spherical shells /9/ and stiffened panels of the type of wide struts /10,11/. The analysis of the equilibrium branches of such systems with the imperfections present only in the first mode was given in /12/. We note that in contrast to the papers mentioned above the coefficients d_i in (3.1) were computed according to the modified local mode except in cases when the displacement over the overall mode does not alter its sign.

Eliminating z_1 from (3.3), we obtain an equation of the projection of the equilibrium branch on the $\lambda - z_2$ plane

$$\left(1-\frac{\lambda}{\lambda_1}\right)\left(1-\frac{\lambda}{\lambda_2}-\frac{\lambda}{\lambda_2}\frac{\bar{z}_2}{z_1}\right)-\frac{1}{2}z_2^2-\frac{\lambda}{\lambda_1}\bar{z}_1=0 \tag{3.5}$$

The value $\lambda = \lambda_*$ for the limit or bifurcation point is determined in the course of eliminating z_2 from (3.4), (3.5)

$$\left(\frac{\lambda_*}{\lambda_2}\right)^2\frac{\lambda_2}{\lambda_1}-\frac{\lambda_*}{\lambda_2}\left(1+\frac{\lambda_2}{\lambda_1}+\bar{z}_1\frac{\lambda_2}{\lambda_1}\right)+1=\frac{3}{2}\left[\left(1-\frac{\lambda_*}{\lambda_1}\right)\frac{\lambda_*}{\lambda_2}\bar{z}_2\right]^{2/3} \tag{3.6}$$

The corresponding displacement over the local mode is

$$z_2=\left[\left(1-\frac{\lambda_*}{\lambda_1}\right)\frac{\lambda_*}{\lambda_2}\bar{z}_2\right]^{1/2} \tag{3.7}$$

The solution of equation (3.6) which yields λ_* as an implicit function of the imperfection parameters $\bar{z}_1$ and $\bar{z}_2$ (the surface of sensitivity to the imperfections) is easily obtained using the method of iterations.

4. Stiffened plate under compression. We consider as an example a problem of stability of a plate stiffened with equally spaced ribs of rectangular cross section, under compression in the direction of the ribs. The loaded edges are assumed to be hinged, and the load-free edges are free (wide strut). In the case of thin ribs the exact description of the linear local modes requires that the ribs be treated as plates. Such a solution was obtained in /11/ in contrast to /10/ et al. where the rib was regarded as a rod. All authors, however, when solving the nonlinear problem, took into account only a single local mode corresponding, as a rule, to the minimum eigenvalue. Below we give the results of the computations, taking into account the set of local modes with critical loads close to the minimum load. The coefficients of (1.9), (1.11) were found from the formulas given in /11/. A plate with the following parameters was considered (Fig.1): $t_1/c_1=0.05$; $t_2/c_2=0.028$; $\omega=t_1c_1/t_2c_2=0.1$; $c_2/L=0.3859$. The dimensionless stress $\sigma^\circ=\sigma\cdot 10^3/E$ (E is the modulus of elasticity) was taken as the load parameter λ. The critical stresses for the local modes σ_M° for various half-wave numbers m along the length are depicted by the solid curve *1* in Fig.2. Two relative minima correspond to the modes with half-wave length of the order of the distance between the stiffeners $(m=3)$ and of stiffener height $(m=7)$. The dashed line *1* depicts the data obtained for the linear problem using the rod scheme for the ribs (in analogy to /10/). Below we give, for various modes, the values of σ_M° and the ratios W_0/W_p where W_0 is the panel displacement amplitude (halfway between the ribs) and W_p is the rib edge displacement

m	2	3	4	5	6	7	8	9
σ_M°	2.940	2.794	3.117	3.006	2,848	2,845	2,955	3.149
W_0/W_p	1.023	0.756	0.279	0.048	0,014	$4.6\cdot10^{-3}$	$1.8\cdot10^{-3}$	$0.8\cdot10^{-3}$
d_1	$-5.0\cdot10^{-3}$	0.058	0.535	1.440	1.600	1.609	1.547	1.448
d_2	$-9.7\cdot10^{-3}$	0.1513	0.529	0,925	1.424	1.964	2.498	2.998
σ_*^0	2.697	2.181	1.578	1.111	0.982	0.933	0.933	0,964

At sufficiently large m we have $W_p \gg W_0$ and the mode can be classified as "local buckling of the stiffener".

In computing the nonlinear case the amplitudes of the initial deflection $\bar{\xi}_1=0.25$ and the equivalent local deflection $\bar{\xi}^\circ=0.25$ were given (the amplitudes are relative to the panel thickness). First the interactions of the overall mode with each of the local modes were considered one by one. The dimensionless limit stresses are shown by the solid line *2* in Fig. 2. The values of the coefficients d_i and limit stresses σ_*° are given above. For comparison, Fig.2 shows the results of the computations carried out according to the scheme using rod-type stiffener (dashed curve *2*). As expected, this approach can be used at small values of m when the displacements of the panel are sufficiently large, but yields erroneous results in the case of "local stiffener buckling". Curve *3* depicts the results of /2/ where an approximate solution was obtained using a provisional model of attachment of the stiffener to the panel. It was assumed that $\bar{\xi}_1=0.5$; $\bar{\xi}_2=0$ (the solution in /2/ takes into account only the overall deflection). This solution was found satisfactory at large m.

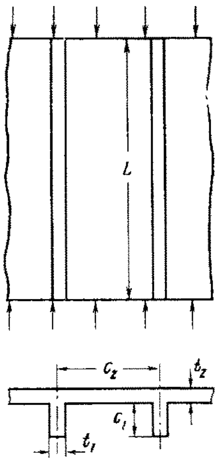

Fig.1

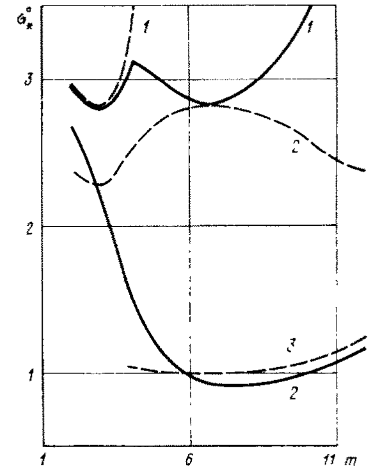

Fig.2

After this we obtain a solution of the complete system of equations (1.9), (1.11) with nine modes $m = 2, 3, \ldots, 10$ taken into account. The initial deflection in each mode was assumed, in accordance with the given value of ξ°, equal to $\xi_s = 1/12$. The limit value σ_*° was found to be equal to 0.9684, which is larger than the value of σ_*° obtained using a single mode $m = 8$ with equivalent deflection (and the adjacent modes $m = 7.9$).

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Translated by L.K.